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Comparative Performance Evaluation of SVD-based Image Compression

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Abstract— **Image compression is an absolute need for current multimedia storage/communication situations. Several image compression algorithms are in daily use. In this paper we consider rank reduction algorithm based on singular value decomposition (RR-SVD), when applied to JPEG coded images. The SVD-based rank reduction algorithm compresses the image by eliminating some of its singular values based on human eye sensitivity. We evaluate the performance of RR-SVD when applied to the whole image in comparison with its block-based application on a number of images in different resolutions and with different compression parameters. We focus on compression ratio (CR), compression time and compressed image quality versus the original image. Our results indicate that block-based application of RR-SVD outperforms whole image application in most cases with a noticeable margin depending on the resolution. Also, we provide suggestions for choosing the optimum operating point based on our results.**

Keywords-component: Singular Value Decomposition; rank reduction; image compression; compression ratio

I. INTRODUCTION

In today's world, multimedia file types such as images and videos are in intensive daily use and their large size would create obstacles in storage and communications. Therefore, optimal compression of such files is of paramount value and importance. If these files were not compressed, the storage\bandwidth needed to store\transfer them would have been several times the current values, which is not desirable at all. While compression is used in many applications such as for data files, audio, video, images etc. Our concern in this paper is compression of images. We focus on rank reduction SVD-based compression method, when apply alongside JPEG compression. We consider three criteria in our comparisons namely compression ratio (CR), computational overhead, indicated by compression time, and compressed image quality in comparison with the original image.

In JPEG algorithm, the image is first divided into 8×8 blocks. Then discrete cosine transform (DCT) is applied to each 8×8 block. Using a quantization matrix, high-frequency components of each block are removed and then entropy coding is performed leading to the compressed image [1]. In the rank reduction SVD-based algorithm the image matrix A is first decomposed by SVD into three matrices, namely $\boldsymbol{U}$, $\boldsymbol{S}$ and $\boldsymbol{V}$, so that $\boldsymbol{A} = \boldsymbol{USV^T}$. Assuming that the image is a $m \times n$ matrix, $\boldsymbol{U}$ will be $m \times m$, $\boldsymbol{V}$ will be $n \times n$ and $\boldsymbol{S}$ will be $m \times n$. $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthogonal matrices and $\boldsymbol{S}$ is a diagonal matrix with diameter values that are the same as the singular values of the image in descending order [2]. Then, the rank reduction algorithm is applied so that k values are chosen from the singular values of the image and the matrices $\boldsymbol{U}$, $\boldsymbol{S}$ and $\boldsymbol{V}$ would become $m \times k$, $k \times k$ and $n \times k$ respectively. Finally, the compressed image would be obtained as $\boldsymbol{B} = \boldsymbol{USV^T}$. In [3] the rank reduction algorithm was applied to the SVD of three low resolution images. Increasing the number of singular values of the image, they compared the results according to three metrics, namely mean square error (MSE), peak signal to noise ratio (PSNR) and CR. In [4] the same algorithm as in [3] was used with root mean square error (RMSE) and signal to noise ratio (SNR), as additional metrics, as well as, PSNR and MSE to evaluate the effect of increasing the rank of the image\matrix. In [5], an image coder based on the SVD and vector quantization was presented. The singular values and singular vectors of the image sub-blocks were computed at the encoder and quantized using a novel variable bit-rate coding scheme. In [6], three different scenarios on rank reduction algorithm based on SVD were compared. The scenarios were: applying the rank reduction algorithm based on SVD to the original image; applying it to the image sub-blocks; and subtracting the mean of the original image before performing the mentioned algorithm. The performance of these scenarios was evaluated using three metrics which were PSNR, CR, and computational overhead. In [7], the rank reduction algorithm based on SVD was applied to a JPEG image. It considered different ranks for the compressed image and investigated the effect of rank on the metrics which were CR, MSE, PSNR and size of the compressed image.

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In this paper, we evaluate the performance of the compression algorithm, including CR, on different block sizes, i.e. $8\times 8, 16\times 16$ and 32×32, as well as the whole image. The images we test on are already JPEG coded and applying the mentioned algorithm leads to a higher CR. Also, we carry out our experiments on several test images in a number of different resolutions to be able to assess the effects of image complexity and resolution on performance.

The rest of this paper is organized as follows. In Section 2, we have a brief description of the mathematics and theories behind SVD-based rank reduction algorithm. In section 3, we discuss the application of the algorithm mentioned in Section 2 to the whole image as well as to 8×8, 16×16 and 32×32 blocks. The implementation, experiments and results for different cases in terms of compression time, and MSE have been reported in Section 4 and Section 5 concludes the paper.

II. SVD OF A MATRIX AND RANK REDUCTION ALGORITHM

Suppose we have an image $\boldsymbol{A}$, where $\boldsymbol{A}$ is a $m\times n$ matrix. We know that symmetric and Hermitian matrices have real eigenvalues and orthogonal eigenvectors and positive semi-definite matrices have positive or zero eigenvalues. For a complex matrix $\boldsymbol{A_{m\times n}}$, $\boldsymbol{AA}^*$ and $\boldsymbol{A}^*\boldsymbol{A}$ are Hermitian and positive semi-definite matrices and have positive or zero eigenvalues. Now if $m<n$, then the roots of the eigenvalues of $\boldsymbol{AA}^*$ are called singular values of $\boldsymbol{A}$ and if $m>n$, then the roots of eigenvalues of $\boldsymbol{A}^*\boldsymbol{A}$ are called singular values of $\boldsymbol{A}$. If the matrix is real, then all the arguments discussed are true if we replace $\boldsymbol{A}^*$ with $\boldsymbol{A^T}$ and Hermitian become symmetric.

One of the matrix decomposition methods is based on SVD. The matrix $\boldsymbol{A_{m\times n}}$ with the rank p can be decomposed as $\boldsymbol{A}=\boldsymbol{USV^T}$ where $\boldsymbol{U_{m\times m}}=[\boldsymbol{u_1} \dots \boldsymbol{u_m}]$ and $\boldsymbol{V_{n\times n}}=[\boldsymbol{v_1} \dots \boldsymbol{v_n}]$ are orthogonal matrices [2, 3, 4, 5, 6, 7, 8, 9, 10]. The columns of $\boldsymbol{U_{m\times m}}$ are composed of eigenvectors of $\boldsymbol{AA^T}$ and the columns of $\boldsymbol{V_{n\times n}}$ are composed of eigenvectors of $\boldsymbol{A^TA}$ [2, 5, 6]. $\boldsymbol{S_{m\times n}}$ is a diagonal matrix [2, 3, 4, 5, 6, 7, 8, 9, 10] whose diameter values are the singular values of $\boldsymbol{A}$ [2, 5, 6, 7, 8, 9, 10].

$$
\begin{aligned}
&S_{m\times n}=diag(\sigma_1,\dots,\sigma_\mu),\\
&\sigma_1\geq\sigma_2\geq\dots\geq\sigma_p>0\,,\\
&\mu=\min\{m,n\},\\
&\sigma_{p+1}=\sigma_{p+2}=\dots=\sigma_\mu=0.
\end{aligned}
\tag{1}
$$

where σ_1 and σ_p are the largest and smallest nonzero singular value of the matrix $\boldsymbol{A}$, respectively [6, 7, 8, 9]. Now we can apply the rank reduction algorithm to the SVD of the matrix $\boldsymbol{A}$, as follows. Consider the matrix $\boldsymbol{A}$ with the rank p. The SVD of it is:

$$
\begin{aligned}
&A=USV^T,\\
&A=[U_1|U_2]\begin{bmatrix}\sigma_1 & \cdots & 0\\ \vdots & \ddots\ \sigma_p & \vdots\\ 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}V_1^T\\ V_2^T\end{bmatrix}.
\end{aligned}
\tag{2}
$$

We need to find the matrix $\boldsymbol{B}$ with the rank k $(k<p)$ so that $||\boldsymbol{A}-\boldsymbol{B}||_2$ becomes small. So equation (2) can also be written as [5, 7, 8, 9]:

$$
\begin{aligned}
A=\ &u_1\sigma_1v_1^T+u_2\sigma_2v_2^T+\dots+u_k\sigma_kv_k^T+u_{k+1}\sigma_{k+1}v_{k+1}^T+\\
&\dots+u_p\sigma_pv_p^T,\\
&\sigma_1\geq\sigma_2\geq\dots\geq\sigma_k\geq\sigma_{k+1}\geq\dots\geq\sigma_p\geq 0.
\end{aligned}
\tag{3}
$$

If the difference between σ_k and σ_{k+1} is high enough, we can ignore other singular values and consider the approximation of $\boldsymbol{A}$ as $\boldsymbol{B}$ [4, 5, 6, 7, 8, 9]:

$$B=u_1\sigma_1v_1^T+u_2\sigma_2v_2^T+\dots+u_k\sigma_kv_k^T. \tag{4}$$

The difference between the two matrices is expressed as:

$$A-B=u_{k+1}\sigma_{k+1}v_{k+1}^T+\dots+u_p\sigma_pv_p^T. \tag{5}$$

The following equation is a SVD:

$$A-B=U_{1b}\begin{bmatrix}\sigma_{k+1} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \sigma_p\end{bmatrix}V_{1b}^T. \tag{6}$$

Note that we have $||\boldsymbol{A}-\boldsymbol{B}||_2\ \leq\sigma_{k+1}$ which is the approximation error. Now that the matrix $\boldsymbol{B}$ is obtained, then $\boldsymbol{U}$ is $m\times k$, $\boldsymbol{V}$ is $n\times k$ and $\boldsymbol{S}$ is $k\times k$ as we will have $\boldsymbol{B}=\boldsymbol{USV^T}$.

III. APPLICATION OF THE PROPOSED ALGORITHM TO VARIOUS BLOCK SIZES

To evaluate the algorithm presented in Section 2, blocks are considered in different sizes. For the first case, the original image is considered as a block, while in other cases, the original image will be split into 8×8, 16×16 and 32×32 blocks. Please note that in this paper we consider $\boldsymbol{A}$ as a symbol for the whole image and $\boldsymbol{A_i}$ as a symbol for the blocks corresponding to the image.

For the whole image, we apply the SVD to the matrix $\boldsymbol{A_{m\times n}}$ with the rank p. As a result, 3 matrices $\boldsymbol{U}$, $\boldsymbol{S}$ and $\boldsymbol{V}$ are created where $\boldsymbol{A}=\boldsymbol{USV^T}$. Then, we apply the rank reduction algorithm to the SVD of the image. The rank of image $(\boldsymbol{A})$ is reduced to k where $k<p$ and the matrices $\boldsymbol{U}$, $\boldsymbol{S}$ and $\boldsymbol{V}$ are transformed into matrices of sizes $m\times k$, $k\times k$ and $n\times k$ respectively. Now, the reduced rank matrix $\boldsymbol{B}$ is obtained so that $\boldsymbol{B}=\boldsymbol{USV^T}$. For block-based application of the algorithm, we first divide $\boldsymbol{A_{m\times n}}$ with the rank p into 8×8, 16×16 or 32×32 blocks. Then we apply the algorithm, similar to the way mentioned above for the whole image, to each and every block.

One should note that the number of 16×16 and 32×32 blocks in an image would be less than 8×8 blocks. On the other hand, the degree of freedom to remove singular values in

16×16 and 32×32 blocks will be higher than 8×8 blocks because the rank of the 16×16 and 32×32 blocks is higher than 8×8 blocks. Therefore, larger blocks are expected to lead to higher CRs.

IV. EXPERIMENTAL RESULTS

In this section, we compare the results of rank reduction algorithm based on SVD and blocks of different sizes. For block sizes, as mentioned before, the whole image, as well as 8×8, 16×16, and 32×32 blocks of the original image are considered. The images used for test are JPEG compressed and further compression is expected by applying the SVD-based rank reduction algorithm. The original images are of high resolution and we use the resize algorithm in MATLAB to also create images at lower resolutions.

The comparative metrics we consider are CR, MSE, and compression time for a rough indication of the computational overhead. For all comparisons, CR has been kept constant. In other words, for each image with different resolutions, the final size of the compressed image is the same and the compression time, and MSE of the two situations are compared. Time is measured by MATLAB. For an $m \times n$ image, MSE is expressed by:

$$MSE = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}(f(i,j) - g(i,j))^2. \quad (7)$$

where f is the original image and g is the compressed image. In all comparisons, two situations are considered. The first one is the application of rank reduction algorithm to the whole image and the other one is applying the same algorithm to various block sizes of the original image. These include 8×8, 16×16, and 32×32 blocks. Also, the experiments are carried out on 5 high-resolution images, while the results are reported for just one of the images (test 1) for brevity and due to the similarity of results obtained for different images. The considered images are shown in Figures 1-5. Also, in the following plots, the resolutions are specified by numbers, according to Table 1.

TABLE 1. LIST OF RESOLUTIONS USED IN TEST 1 EVALUATIONS

Number	resolution
1	914x671
2	3655x2681
3	5482x4022
4	7309x5362

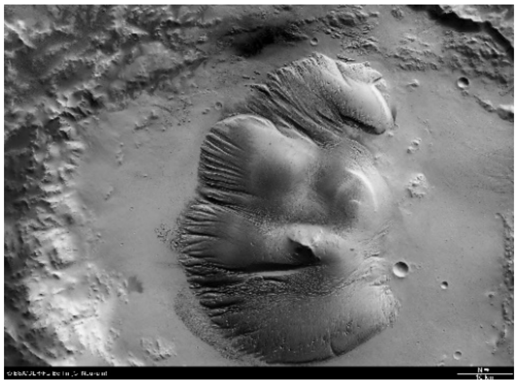


Figure 1. Test 1 (7309×5362)

Figure 2. Test 2 (2000×3008)

Figure 3. Test 3 (3476×2317)

Figure 4. Test 4 (1920×1200)

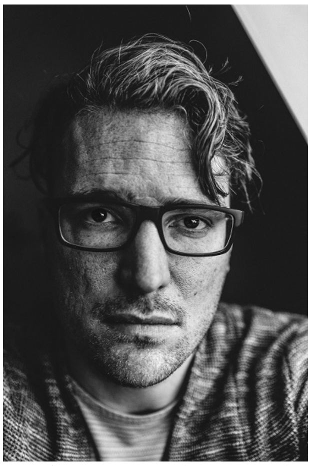
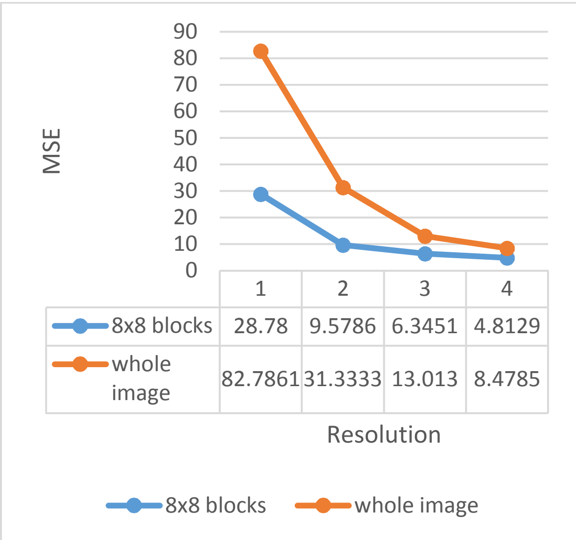
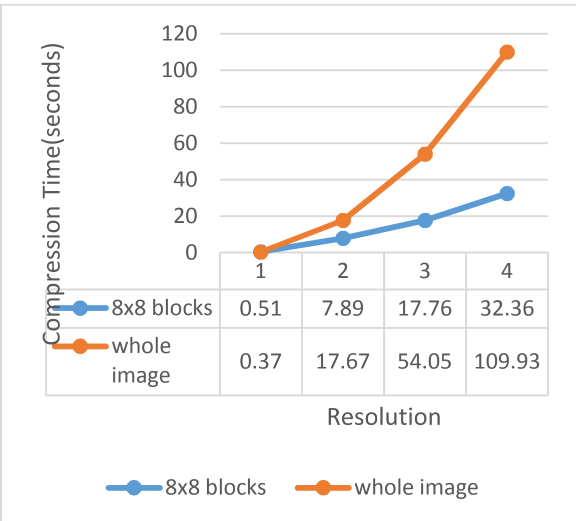


Figure 5. Test 5 (4016×6016)



(a)

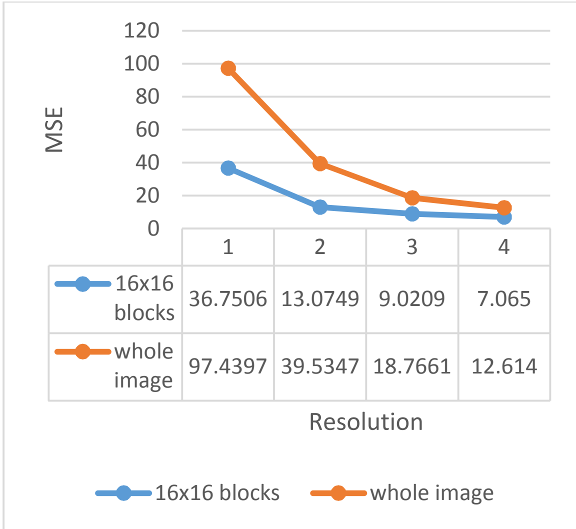


(b)

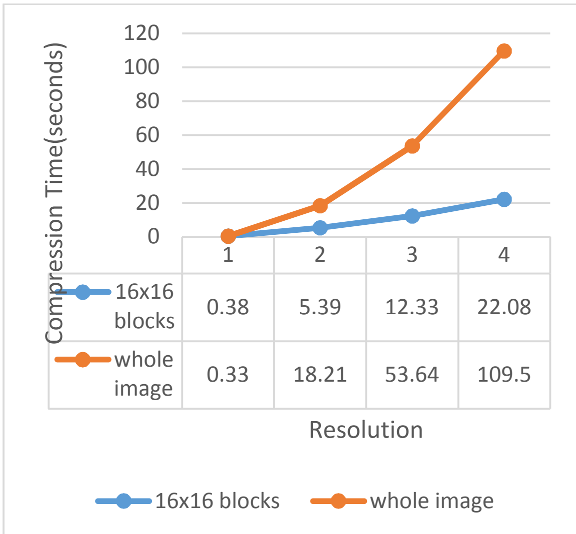
Figure 6. Test 1 evaluations with an 8×8 block size for 4 different resolutions, a) MSE; b) compression time.

Figure 6 includes the results of applying the algorithm to the first figure (Test 1) with a block size of 8×8. We keep only two singular values for each 8×8 block. Using the same CR for both situations, we calculate and apply the number of singular values that should be eliminated from the whole image case, shown in orange in our graphs.

Two similar sets of experiments are also carried out on block sizes of 16×16 and 32×32 and the results depicted in Figures 7 and 8, respectively. Please note that for the cases of 16×16 and 32×32 blocks, only three and four singular values for each block are retained, respectively.



(a)



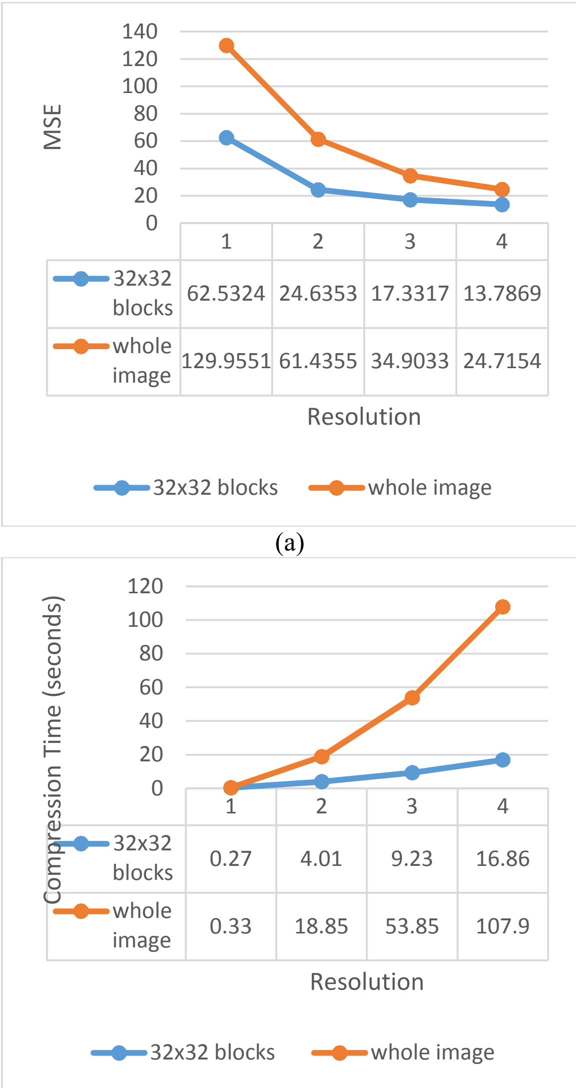
(b)

Figure 7. Test 1 evaluations with a 16×16 block size for 4 different resolutions, a) MSE; b) compression time.

As can be seen in all three images, the application of the SVD-based algorithm to blocks of any size (8×8, 16×16 and 32×32) has led to a superior quality in comparison with

its application to the whole image. This can be seen in all resolutions, but is even clearer in lower resolutions.

Regarding the compression time, once again the application of the algorithm to the blocks of the image has been far less time consuming, except for the case of the lowest resolution, where the two cases do not show significant differences.



(a)

(b)

Figure 8. Test 1 evaluations with a 32×32 block size for 4 different resolutions, a) MSE; b) compression time.

The above results illustrate the fact that once the rank reduction algorithm is applied to the blocks of the original image, almost in all cases, both MSE and compression time are improved.

In another set of experiments, we evaluate the change of MSE versus CR, while keeping different numbers of singular values of each block in each situation (8×8 , 16×16 and 32×32 blocks of the original image). Our results indicate the existence of separate regions with different behaviors in such plots. We call them regions 1 to 4. These regions are expected due to the form of the variation of MSE and CR by preserving different numbers of singular values. In region 1, the variation of MSE versus CR is totally irregular. In region 2 this variation is similar to square root function, while in regions 3 and 4 the variations are similar to exponential and linear functions respectively. The occurrence of each region is not associated with the index we have assigned to it. In other words, we do not expect the region 1 to occurs first and then region 2 and so on. Another point to note is the number of occurrences of regions for each block size. In general, there can be four regions for each block size, while it is even possible that a particular region occurs more than once for each block size. The only exception is that region 1 can only occur in the first step and there is no chance of its occurrence in the next steps. These results are shown in Figures 9 to 11. The results are reported for just one of the images (test 1) for brevity.

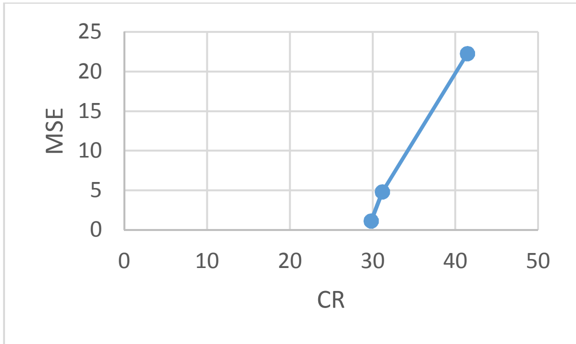
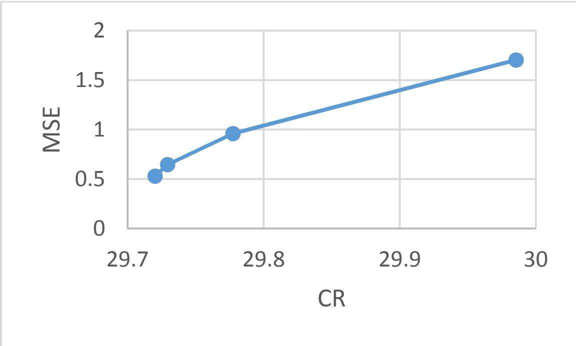


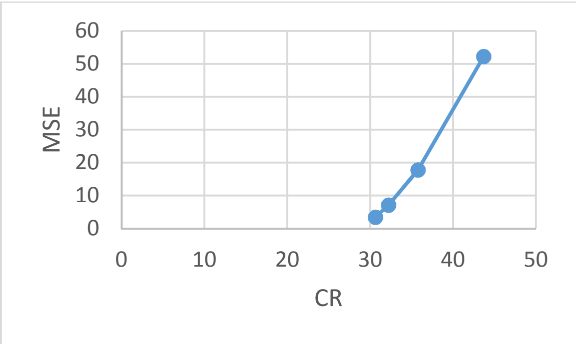
Figure 9. MSE versus CR curve in region 2 for Test 1 image with a block size of 8×8.

As can be seen, at different sizes of the block for test 1, in MSE vs. CR curve, region 4 is not seen. Due to the different paces of change of MSE versus CR in Figures 9, 10(a) and 11(a) compared to Figures 10(b) and 11(b), careful inspection is required before deciding on the working points in such figures. A rational view would be to choose the starting point of the exponential region, as further on the MSE increases sharply with increase in CR, while before that the rate of change is small.

On the other hand, if, regardless of the CR value, the least MSE is required, one can even move to the start of region 2, as it leads to further reduction of MSE, with little decrease in CR, except in Figure 9, where the change in both MSE and CR are noticeable. For tests 2 and 3, the results are quite similar to test 1. For tests 4 and 5, the results are not exactly the same as test 1, but the analysis of the suitable region for compression is similar to test 1. Depending on the goal of image compression for the users, either region 2, 3 or 4 are recommended. In any case, the use of region 1 is not recommended, as the results are not predictable in that region.

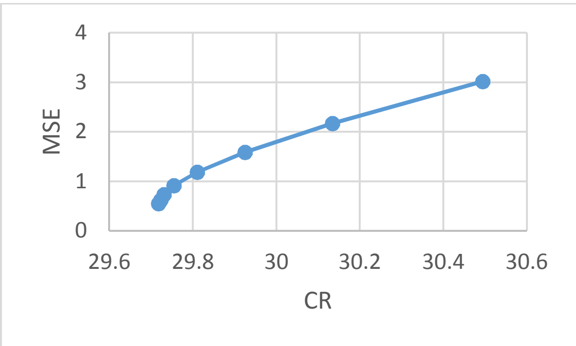


(a)

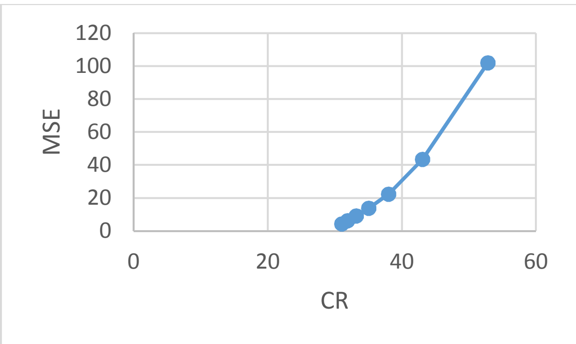


(b)

Figure 10. MSE versus CR curves for Test 1 image with a block size of 16 × 16 in a) region 2; b) region 3.



(a)



(b)

Figure 11. MSE versus CR curves for Test 1 image with a block size of 32 × 32 in a) region 2; b) region 3.

V. Conclusion

In this paper we carried out several experiments using SVD-based image compression. Our results indicate that applying rank reduction algorithm to the SVD of various block sizes of image has led to a superior quality in comparison with its application to the whole image. Furthermore, the application of the algorithm to the blocks of image has been far less time consuming, except for the case of the lowest resolution, where the two cases do not show significant differences. Evaluation of the changes of MSE versus CR, while keeping different numbers of singular values of each block in each situation (8×8 , 16×16 and 32×32 blocks of the original image), has led to separate regions with different behaviors. In region 1, the variation of MSE versus CR has been found to be totally irregular, while in regions 2, 3 and 4 this variation is similar to square root, exponential and linear functions, respectively. Depending on the goal of image compression for the users, either region 2, 3 or 4 are recommended as the working region, while region 1 has been found inappropriate due to the irregularity of the change of MSE versus CR in this region.

IV. REFRENCES

[1] G.K. Wallace, "The JPEG Still Picture Compression Standard," *IEEE Transactions on Consumer Electronics*, vol. 38, no. 1, pp. 1-17, 1992.

[2] G. Strang, *Linear Algebra and its Applications*, Thomson Learning, 1988.

[3] H.S. Prasantha, H.L. Shashidhara, K.N.B. Murthy, "Image compression using SVD," in *Proc. International Conference on Computational Intelligence and Multimedia Applications*, Sivakasi, Tamil Nadu, India, Dec. 2007, pp. 143-145.

[4] R. Samnotra, R. Singh, J. Khan, "Image Compression Using SVD Technique and Measurement of Quality Parameters," *International Journal of Engineering and Computer Science ISSN:2319-7242*, vol. 2, no. 12, pp. 3484-3487, 2013.

[5] C.S. Mc Goldrick, W.J. Dowling, A. Bury, "Image Coding Using the Singular Value Decomposition and Vector Quantization," *Fifth International Conference on Image Processing and its Applications*, Edinburgh, UK, July. 1995, pp. 296-300.

[6] M. Tian, S-W. Luo, L-Z. Liao, "An Investigation into Using Singular Value Decomposition as a Method of Image Compression," in *Proc. International Conference on Machine Learning and Cybernetics*, Guangzhou, China, China, Aug. 2005, pp. 5200-5204.

[7] K.M. Aishwarya, R. Ramesh, P.M. Sobarad, V. Singh, "Lossy Image Compression using SVD Coding Algorithm," in *Proc. International Conference on Wireless Communications, Signal Processing and Networking (WiSPNET)*, Chennai, India, March. 2016, pp. 1384-1389.

[8] M.S. Kahu, M.R. Rahate, "Image Compression using Singular Value Decomposition," International Journal of Advancements in Research and Technology, vol. 2, Issue 8, pp. 244-248, 2013.

[9] Vinita Cheepurupalli, Sierra Tubbs, Khadijah Boykin, Naima Naheed, "Comparison of SVD and FFT in Image Compression," in *Proc. International Conference on Computational Science and Computational Intelligence (CSCI),* Las Vegas, NV, USA, Dec. 2015, pp. 526-530.

[10] Allaeldien Mohamed G. Hnesh, Hasan Demirel, "DWT-DCT-SVD based Hybrid Lossy Image Compression Technique," in *Proc. International Conference on Image Processing, Applications and Systems (IPAS),* Hammamet, Tunisia, Nov. 2016, pp. 1-5.